Binomial [77 marks]

1. [Maximum mark: 4]

Find the coefficient of x^8 in the expansion of $(2x - 5)^{11}$. [4]

2. [Maximum mark: 4]

Find the coefficient of x^6 in the expansion of $(2x - 5)^9$. [4]

3. [Maximum mark: 6]

The binomial expansion of $(1 + kx)^n$ is given by
$1 + \frac{9x}{2} + 15k^2x^2 + \ldots + k^nx^n$, where $n \in \mathbb{Z}^+$ and $k \in \mathbb{Q}$.

Find the value of *n* and the value of *k*. [6]

4. [Maximum mark: 6]

The binomial expansion of $(1 + kx)^n$ is given by
$1 + 12x + 28k^2x^2 + \ldots + k^nx^n$ where $n \in \mathbb{Z}^+$ and $k \in \mathbb{Q}$.

Find the value of n and the value of k. [6]

5. [Maximum mark: 7]

The coefficient of x^6 in the expansion of $\left(ax^3 + b\right)^8$ is 448.

The coefficient of x^6 in the expansion of $\left(ax^3 + b\right)^{10}$ is 2880.

Find the value of a and the value of b, where $a, b > 0$. [7]

6. [Maximum mark: 6]

Consider the expansion of $\frac{(ax+1)^9}{21x^2}$, where $a \neq 0$. The coefficient of the term in x^4 is $\frac{8}{7}a^5$.

Find the value of a. [6]

7. [Maximum mark: 15]

(a.i) Expand and simplify $(1-a)^3$ in ascending powers of a. [2]

(a.ii) By using a suitable substitution for a, show that

$$1 - 3\cos 2x + 3\cos^2 2x - \cos^3 2x = 8\sin^6 x.$$ [4]

Consider $f(x) = 4\cos x\left(1 - 3\cos 2x + 3\cos^2 2x - \cos^3 2x\right)$.

(b.i) Show that $\int_0^m f(x)\,\mathrm{d}x = \frac{32}{7}\sin^7 m$, where m is a positive real constant. [4]

(b.ii) It is given that $\int_m^{\frac{\pi}{2}} f(x)\,\mathrm{d}x = \frac{127}{28}$, where $0 \leq m \leq \frac{\pi}{2}$.
Find the value of m. [5]

8. [Maximum mark: 7]

Consider the binomial expansion
$(x+1)^7 = x^7 + ax^6 + bx^5 + 35x^4 + \ldots + 1$ where $x \neq 0$ and $a,\ b \in \mathbb{Z}^+$.

(a) Show that $b = 21$. [2]

(b) The third term in the expansion is the mean of the second term and the fourth term in the expansion.

Find the possible values of x. [5]

9. [Maximum mark: 5]

In the expansion of $(x + k)^7$, where $k \in \mathbb{R}$, the coefficient of the term in x^5 is 63.

Find the possible values of k. [5]

10. [Maximum mark: 5]

Consider the expansion of $\left(3 + x^2\right)^{n+1}$, where $n \in \mathbb{Z}^+$.

Given that the coefficient of x^4 is $20\,412$, find the value of n. [5]

11. [Maximum mark: 6]

Consider the expansion of $\left(3x^2 - \frac{k}{x}\right)^9$, where $k > 0$.

The coefficient of the term in x^6 is 6048. Find the value of k. [6]

12. [Maximum mark: 6]

Consider $\binom{11}{a} = \frac{11!}{a!\,9!}$.

(a) Find the value of a. [2]

(b) Hence or otherwise find the coefficient of the term in x^9 in the expansion of $(x + 3)^{11}$. [4]

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